Book Reviews

The Economics of Input–Output Analysis
Thijs ten Raa

Although it carries a different title, this book is essentially a second edition of the author’s Linear Analysis of Competitive Economies (ten Raa, 1995). There is a good deal of overlap. Both works contain 14 chapters, and while chapter titles (usually) differ between the two works their respective subject matter is very much the same – although sometimes updated or expanded in the newer volume.\(^1\) The 1995 book was reviewed in this Journal (Rose, 1996); the bulk of that review, to which the reader is referred, remains relevant to the new volume.

In the opening paragraph of his Preface, the author sets his agenda. He suggests (p. xi) that input–output analysis is the main tool to help answer three key questions regarding the economy as a whole: what is its performance (efficiency, productivity growth); what is its comparative advantage (relative to the rest of the world); and what are the effects of environmental constraints? Addressing these questions in a consistent framework is a major accomplishment of this book. Put differently (as the author does in the Preface, again p. xi) his goal is to undermine the perception that input–output analysis is a ‘rather mechanical tool not easily applicable to free market economies with competitive valuations.’

The first four chapters lay the foundations, covering a number of basics, although not always in standard ways. Chapter 1 presents the economic problem as one of ‘... maximization of some objective subject to constraints’ (p.1). Next come mathematical preliminaries (functions, differentiation and integration), constrained maximization, some matrix notation and brief introductions to linear programming and input–output analysis.

This chapter also includes the first illustration of what will turn out to be a recurring theme: the incorporation of an input–output material balance constraint (here \(y \leq x - Ax\)) into a linear programming framework. Given an economy’s available stocks of capital, \(K\), and labor, \(L\), with associated vectors of capital and labor input coefficients, the factor constraints \(K'x \leq K\) and \(L'x \leq L\), along with non-negativity requirements, \(x \geq 0\), generate the linear program

\[
\max_{xy} p'y \text{ subject to } y \leq x - Ax, K'x \leq K, L'x \leq L \text{ and } x \geq 0
\]

The objective is to maximize some valuation of final demand, where \(p\) is some vector of prices.\(^2\) All this in less than 12 pages of text. Chapters 2–4 then elaborate on input–output
basics, multiplier analysis (two pages on Miyazawa inverses are new to this volume) and linear programming in more detail.

Use \((U)\) and make \((V)\) tables from the newer commodity-industry accounting framework are introduced in Chapter 5, and the economy’s net output vector becomes \((V' - U)i\), where \(i\) is the summation vector (column of 1s). This is the modern version of \(x - Zi\), or \(x - Ax\), in pre-commodity-industry accounting terms (where \(Z\) is the transactions matrix). Chapter 6 contains a discussion of systems of national accounts (SNAs) and the positioning of the make and use data for an economy within the SNA framework. It also includes a discussion of value-added taxes and how they can be imputed back to producing sectors in an economy with the help, in part, of \(U\) and \(V\). Finally, there is a brief (one-page) discussion of social accounting matrices (SAMs), new to this volume. Chapter 7 lays out alternative ways of creating technical coefficients from the make and use matrices, invoking commodity technology, \(A(U, V)_c = U(V)^{-1}\), industry technology, \(A(U, V)_i = U((Vi)^{-1})V((Vi)^{-1})\), and various mixed technology assumptions. These constructs are examined in relation to a set of four ‘desirable properties’: material balance, financial balance, price invariance, and scale invariance. Only commodity technology coefficients satisfy all four.

In Chapter 8, the macroeconomic production function of an economy is derived via a linear program in which the objective is to maximize value added for the economy, subject to labor and capital constraints. Value added is a function of the net output matrix, \((V' - U)\), which brings input–output data into the picture. Chapter 9 addresses macroeconomic (in)efficiency (question 1 from the Preface) in a linear programming model in which an economy’s level of consumption is maximized subject to upper limits on factor use and also an upper limit on the consumption vector that is set by available net output, namely

\[
\max_{c,s} i'y_c \text{ subject to } (V' - U)s \geq yc, k's \leq K, \text{ and non-negativities}
\]

where \(y\) is the current consumption vector, \(c\) is the expansion factor, \(K\) and \(k\) represent endowments and uses of all factors and \(s\) contains relative sector activity levels. \(^4\) Illustrations for both the Canadian and Dutch economies are discussed. (The Dutch example, one page, is new to this volume.) Chapter 10 presents a similar kind of linear programming structure to investigate trade between two countries (the second question from the Preface) – in particular where the objective, again, is maximization of consumption. There is a very brief summary of an application involving Canada and Europe (ten Raa and Mohnen, 2001). Chapter 11 briefly covers environmental issues (question 3), namely energy and pollution. Another version of the basic linear programming model is sketched, this time with accounts for pollution (bads) included in the make and use matrices. There is a final section (new, one page) on globalization, in which pollution is decomposed into three (additive) causes: domestic consumption, pollution due to exports and pollution due to abatement.

Chapter 12 looks at total factor productivity (TFP) growth and its decomposition into three additive components: technical change (changes in input–output coefficients, from \(U\) and \(V\)), changes in terms of trade and changes in efficiency. A discussion of a study for Canada is new (ten Raa and Mohnen, 2002) as are discussions of international spillovers and intersectoral spillovers (Shestalova, 2001, and ten Raa and Wolff, 2000,
respectively). The author concludes (p. 158): ‘The macroeconomic concept of TFP growth has been grounded in changes of microeconomic fundamentals.’

Chapter 13 introduces the dynamic input–output model, namely \( x = Ax + B \Delta x + y \) and then goes on to develop a dynamic analogue of the material balance conditions, based on the author’s work with ‘convolution products.’ Chapter 14 is a very brief (six pages plus references) introduction to stochastic input–output modeling, including a (new) discussion of an application to Andalusian data (ten Raa and Rueda-Cantuche, 2004) that examines the bias in multipliers that are based on published make and use matrices.

There are a few puzzlements. References are made in passing to ‘an event with zero measure’ (p. 48) and a ‘sub-set of measure zero’ (p. 105); these seem to imply an unexpected mathematical sophistication on the part of the reader. The Hawkins–Simon conditions are described as follows: ‘A less practical but general test [for the existence of the Leontief inverse] is the positivity of the so-called principal minors of \( A \) (Hawkins and Simon, 1949).’ I thought that for something like 50 years the Hawkins–Simon conditions had been presented as positivity conditions on the principal minors of \((I - A)\). Of course the two are related, but this seems to invite confusion.

As for the potential audience, the author ‘...target[s] advanced undergraduate or new graduate students of economics who do not panic when a function is differentiated or integrated’ (p. xii). Later on that page, he suggests (reasonably) that the book might serve ‘...as a reference source, particularly by applied equilibrium economists and national accountants using input–output measures.’ Certainly it is indeed a valuable reference to anyone aware of input–output basics. It does not seem to me that it would serve as well in the role of a student ‘introduction’ to input–output analysis. I would think of it rather as an accompaniment to a more complete introductory text, but that is of course a matter of taste.

Thijs ten Raa recently described one of his research interests as ‘...reconcil[ing] input–output analysis and neoclassical economics.’ This book very amply illustrates the scope of his considerable work on that issue over the last decade or so. Many of the updated chapters in this book are enriched by the author’s recent (post-1995) research – usually taking the form of numerical applications of the ideas that are presented in the new work.

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Notes

1This despite the fact that the new volume contains only three more pages of text than the former – 182 versus 179 at 42 (new) and 44 (old) lines per page.
2As Rose pointed out in his review of the earlier edition of this book (Rose, 1996, p. 417), ‘...even Leontief (1986) has used an IO optimizing framework’. The reference is to a linear program with the input–output material balance conditions as constraints in Leontief (1986, Ch. 19).
3This material draws heavily on Kop Jansen and ten Raa (1990).
4Relative to the base (observation) period, when all output levels are unity.
5Since, for example, Dorfman et al. (1958, Section 9-2-2). There may be earlier references.
6Adam Rose, commenting on the compact introduction to linear economic models in some 30 pages of Chapters 1–3 of the original volume, suggests that ‘Those who are mathematically inclined, or already
well-versed in economic theory, will find this a great service; others will have to wade through more than 100 pages of a conventional text' (Rose, 1996, p. 416). To build on Rose’s wading thought, a beginner can learn to swim by jumping into deep water over his head, or he can wade in to waist deep before lying down and starting to kick – again, a matter of taste.

Report of a conversation with Wassily Leontief, who is said to have replied ‘Should be easy’ (see ten Raa, 2004, p. 151).

References

Frontiers in Applied General Equilibrium Modeling
Timothy J. Kehoe, T.N. Srinivasan & John Whalley (Eds)

This is a book in honor of Herbert Scarf and his contributions to economics, especially in the field of applied general equilibrium (AGE) modelling. In this field, Scarf’s contributions were crucial, thanks to his algorithm based on simplicial subdivisions (Scarf, 1967). Scarf was the PhD advisor for two of the three editors – T.J. Kehoe (University of Minnesota and Federal Reserve Bank of Minneapolis) and J. Whalley (University of Western Ontario) – while T.N. Srinivasan (Yale University) has worked with Scarf since 1980, including jointly teaching courses on applied general equilibrium.

AGE approaches have been used extensively since the 1980s to analyze economic policies for both developed and developing countries. AGE models are a set of modeling instruments and tools for the evaluation of public policies and other comparative static exercises. AGE modeling is especially attractive for policy makers because it can be used to measure the effects of a specific decision not only on the productive sectors but
also on the remaining agents in an economy. In this way, AGE models can capture the
effects of an exogenous shock on each of the agents, the markets and on the remainder
of the economy.

This volume consists of 15 chapters, written by many of the most important applied
general equilibrium modelers, divided into six parts: general equilibrium theory; computa-
tional methods; macroeconomics and finance; public finance, development, and climate
change; general equilibrium restrictions and estimation of hedonic models; and policy uses
and performance of AGE models. All but two of the chapters were presented at a confer-
ence held at Yale University in April 2002.

The authors present some traditional as well as some newer topics in the field, including
non-convexities in economy-wide models, tax policies, developmental modeling, and
energy modeling. In addition we can find a range of new approaches, conceptual issues,
and computational algorithms (such as calibration), and new areas of application (such
as the macroeconomics of real business cycles and finance). The first chapter, written
by the editors, is very interesting since it provides an overview of the future of this dis-
cipline, starting with a discussion of the origins of applied general equilibrium modeling
and Herbert Scarf’s contributions.

Part 1 has two chapters. The first, by K. Arrow (Stanford University and Nobel Prize
laureate), presents as a personal reflection a recapitulation of the history of general equili-
brium and its use in AGE. The second chapter, by H. Scarf (Yale University) and
C. Wilson (New York University), develops a demonstration of the uniqueness of equili-
brium in the multicountry Ricardo model, under the sufficient condition of gross substi-
tution in aggregate demand.

Part 2 consists of two chapters about computation methods. K. Judd (Hoover Institution)
presents an alternative algorithm for solving dynamic stochastic models, and M. Ferris
(University of Wisconsin at Madison), S. Dirkse (GAMS Development Corporation)
and A. Meeraus (GAMS Development Corporation) discuss a new set of methods for
solving problems, combining facets of optimization and complementarity (mathematical
programs with equilibrium constraints, MPEC).

Part 3 is composed of four chapters dedicated to applications in macroeconomics and
finance. It starts with a chapter by E. Prescott (Arizona State University and Minneapolis
Federal Reserve Bank) where he reviews the role of non-convexities at the micro level in
macro business cycles. L. Ljungqvist (Stockholm School of Economics, CEPR and IZA)
and T. Sargent (New York University and Hoover Institution) both complement and chal-
lenge the results of the previous chapter. The chapter by M. Nakajima (University of Illi-
nois at Urbana-Champaign) and J.V. Rios-Rull (University of Pennsylvania) presents an
extension of previous work. These authors include aggregate real shocks in the economic
activities in the benchmark case, inducing aggregate fluctuations in storage economies.
Finally, A. Araujo (Instituto de Matematica Pura e Aplicada and Fundacao Getulio
Vargas) and M. Páscoa (Universidade Nova Lisboa) model default penalties and collateral
and credit restrictions.

Part 4 describes three applications of AGE to public finance, development and climate
change. In the first one, D.W. Jorgenson (Harvard University and Kennedy School of
Government) and K. Yun (Yonsei University) present a dynamic AGE of the US
economy to assess alternative fiscal policies. In the second application, F. Bourguignon
(World Bank), A.S. Robilliard (DIAL and IRD) and S. Robinson (University of Sussex
and IFPRI) describe a methodology to link microsimulation with a sectoral AGE
model, to analyze issues of income distribution, labor supply and consumption behavior. This part ends with a multiregion, multiperiod, and multisector AGE to describe controversial issues in the debate over the United Nations Framework on Climate Change of 1992 and the Kyoto Protocol of 1997, presented by A.S. Manne (Stanford University).

Part 5 consists of only one chapter, by J. Heckman (University of Chicago and Nobel Prize laureate), R. Matzkin (Northwestern University) and L. Nesheim (University College London and Institute for Fiscal Studies). These authors review the problem of estimating hedonic models with price-differentiated goods and services, using an equilibrium framework.

Finally, Part 6 presents policy uses and performance of AGE models. T.J. Kehoe presents an ex-post evaluation of the results of three multisectoral static AGE models that were developed to assess (ex-ante) the impacts of introducing NAFTA. L. Abrego (IMF) and J. Whalley describe recent applications of general equilibrium computational methods in the international trade area, which are focused backwards (ex-post) rather than forward (ex-ante). S. Devarajan (World Bank) and S. Robinson review the experiences with the use of applied general equilibrium models to affect public policy: structural adjustment policies, international trade, public finance, agriculture, income distribution, and energy and environmental policy.

On the cover of the book, J.B. Shoven (Stanford University) writes that ‘...this book is not only a tribute to Herbert Scarf, the father of general equilibrium computation in economics and one of the greatest economists of the past fifty years, but it is also a valuable handbook on the developments in applied general equilibrium modelling over the last thirty years’. Indeed, a handbook – with a very good collection of papers – that allows the reader to appreciate the importance of this discipline. It is a valuable volume for anyone interested in general equilibrium modeling and its applications. As expressed by R. Boadway (Queens University) on the cover: ‘...for the curious as well as converted’.

Some minor critical points are the following. First, I do miss a chapter dedicated to the compilation issues of the database necessary to develop an AGE model, the Social Accounting Matrix (SAM). Although the last chapter provides some information, I feel it is too limited and the topic requires a more in-depth treatment. Second, Chapters 3 (Macroeconomic and Finance) and 5 (General Equilibrium Restrictions and Estimation of Hedonic Models) do not really comply with the title of the book (i.e. ‘Applied General Equilibrium Modeling’), only to ‘General Equilibrium Modeling’.

In any case, I am convinced that if a book like this had been available at the time I was starting my PhD dissertation, it would have been an invaluable tool in developing my view of the field and in getting the details right. No doubt it would have sped up the first stages of my research.

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References
Near Equilibrium – A Research Report on Cyclic Growth
András Bródy

In the 1970s, a group of NYU students (Chakraborty, Morris, Viet and I) read and discussed András Bródy’s Proportions, Prices, and Planning: A Mathematical Restatement of the Labor Theory of Value. Ever since, Bródy is on my radar screen and I have looked forward to his recent Near Equilibrium, the book that consolidates ten years of fundamental research.

The centerpiece of economics is the theory of value, which attempts to explain the quantities and prices of transactions. There are competing variants. Ricardo and Marx reduce value to labor services. Sraffa and von Neumann use a standard commodity bundle. Smith and Walras bring in demand as a co-determinant. All authors explain the ‘natural’ state of the economy, which means equilibrium or at least steady state growth. Marx’s analysis of the capitalist dynamics is the exception, but it is imprecise and Bródy’s aforementioned restatement focuses on the stable relationships. What remains to be explained are the fluctuations of prices and quantities.

The Walras’ tâtonnement comes close to a dynamic theory of value, but its objective is to explain a tendency towards equilibrium, whereas Bródy argues at length and convincingly that the natural state of an economy, be it free-market, centrally planned, or mixed, is not equilibrium, but fluctuation. In his view, business cycles are not transient but persistent. Fluctuations are intrinsic to the economic system and ought to be explained head-on on the basis of the fundamentals, where the latter are represented by the input–output coefficients of the flows and the stocks in production and consumption.

Bródy fills this gap in the history of economic thought in a subtle manner. Let me introduce his theory via the Walras’ tâtonnement, \( \dot{p} = z(p) \), where \( p \) is the price vector, and \( z \) the excess demand function (demand minus supply). Here \( p \) is not necessarily the equilibrium price (for which excess demand is zero) and, therefore, supply and demand may be off-equilibrium as well. Now if this is the case, a gradual adjustment of price takes place (by the auctioneer). The quantities, however, are immediately in tune with the prices. Producers select inputs and outputs which instantaneously maximize profit. The author draws our attention to the distinction between levels, changes, and rates of variables. For example, it makes a difference if one maximizes profit, moves in the most profitable direction, or maximizes present value. These differences plague Walras’ tâtonnement. Prices are changed according to market conditions, but quantities are set at the right level. Bródy argues, again at length and convincingly, that quantities should not be treated asymmetrically. Quantities adjust. It takes time to install machinery and equipment and to hire and fire workers. Implicitly, Walras assumes that prices are more sluggish than quantities. His auctioneer adjusts prices, but producers and consumers instantaneously reformulate their supplies and demands. Bródy says: let excess supply (\(-z\)) adjust in the direction of price: \( -\dot{z} = p \). He thus replaces the single first-order differential equation by a pair of coupled differential equations.

His insight kills two birds with one stone. First, it resolves the outstanding issue in the theory of value that supply (and hence price adjustment) is ill-defined in the
important case of constant returns to scale. If the price of output is greater than,
smaller than, or equal to the price of the input bundle, supply is infinite, zero, or
indeterminate, respectively. This situation cannot be described by a supply function.
Strictly speaking, the tâtonnement process requires diminishing returns to scale,
which is a big limitation. How does Bródy’s approach solve this problem? In its
simplest form, his model runs as follows. The quantities supplied are given by
vector $x$. Let $A$ be the matrix of input–output coefficients, including the (consump-
tion) coefficients of the households. Then the quantities demanded are $Ax$. The
prices are given by vector $p$. Hence, the costs are $A'p$, where the prime denotes
transposition. Prices adjust in response to excess demand: $\dot{p} = Ax - x$, and quantities
adjust in response to profitability: $\dot{x} = p - A'p$. This is ingenious. Unlike supply, the
direction of profit is well defined for economies with constant returns to scale.
Moreover, the coupling of the price and quantity systems can be seen as a fix of
input–output analysis.

The symmetric treatment of prices and quantities has dramatic impact. I come to the
second bird Bródy kills. The convergence of Walras’ tâtonnement to equilibrium – at
least under weak gross substitutability as shown by Arrow and Hahn – is replaced by per-
sistent cycling around the equilibrium. Indeed, the anti-symmetry in the coupled adjust-
ment process generates only cyclic solutions besides the steady state growth path. The
spectrum of frequencies is determined by the $A$-matrix.

Bródy presents and analyzes two important variations on the basic model. One is
the replacement of derivatives by elasticities. It is reasonable to assume that excess
demand and profitability rates propel entrepreneurs or planners. The linear differential
equations become nonlinear; the consequent Lotka–Volterra equations are ana-
lyzed and found to produce similar cyclic solutions. The other variation is the
introduction of Leontief’s stock coefficients. As in Bródy (1974), he assumes a
relationship between flow and stock coefficients (via the turnover period), a con-
dition that I gave a theoretical foundation in my Structural Economics book (ten
Raa, 2004).

Applying the analysis to the US economy, Bródy finds six cycles, namely the sea-
sonal cycle, a five-year inventory cycle, a diminishing seven-year cycle, a 19-year
Kuznets cycle, a 27-year demographic cycle and 60 + years Kondratiev wave. The
spectrum is the same for the two input–output structures he investigates, namely the
1947 and 1958 Harvard Economic Research Project tables. He also compares to
the actual performance of the US economy and finds that the model explains half of
the fluctuations.

In the third part, the author shares his insights in the methodology of science. As in
physics, the causal model is equivalent to a teleological approach. In fact, he uncovers
the objective function of which the minimization replicates the dynamics of the price-
quantity adjustment process; it is Sraffa’s stipulation of the discrepancy between
nominal surplus (paper profits) and their accumulation.

The author is an artist. With broad brushes he paints the quintessence of the theory of
value – prices, quantities, and their fluctuations – and shows their applicability. I miss
some mathematical and econometric detail, but have the feeling the results are correct.
This book is a goldmine of ideas. With a single, elegant principle – the symmetric treat-
ment of prices and quantities – he unifies the theories of value, growth and the business
cycle.
András Bródy is an independent thinker who has not been treated too well by the political and scientific authorities under communism or after. Yet he has superseded himself. *Near Equilibrium* makes a lasting imprint.

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References
